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Experimental single-parameter temperature and magnetic field conductance function

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Abstract. It is shown that the temperature dependent magnetoresistance data of a sample in the weak antilocalization regime lies on a single curve of d ln $R(B, T)$/d ln B versus ln $(R(B, T)/R(0, T))$. $R(B, T)$ is the resistivity of the sample, B is the applied magnetic field and T is temperature. This result is consistent with the single-parameter zero temperature scaling theory of conduction. It also suggests that, in the presence of a magnetic field and strong spin–orbit scattering, a single-parameter scaling type function is adequate in describing the resistivity.

1. Introduction

Davies *et al* (1983) tried to extend the zero temperature scaling ideas of Abrahams *et al* (1979)—referred to as AALR hereafter—to finite temperatures, in an attempt to determine the AALR scaling function $\beta = \mathrm{d}\ln g/\mathrm{d}\ln L$ from experimental temperature coefficient of resistance (TCR) data. Here g is the *conductance* of a subsystem of arbitrary size L. As they were not successful it was concluded by Davies *et al* (1983) that the single-parameter scaling ideas of AALR were irreconcilable with experiment. White and McLachlan (1986) reconsidered the ideas of Davies *et al* (1983) and succeeded in reconciling the temperature dependent 'scaling' function of Davies *et al* (1983) with the single-parameter AALR scaling function by utilizing an alternative normalization procedure. In this paper it is shown, in both bismuth-based weakly antilocalized systems and systems lying in the hopping regime, that it is possible to obtain the AALR β function from experimental TCR data. This is especially surprising in a strongly spin–orbit coupled system, such as bismuth, where doubt has been expressed (McKinnon 1985 and the references therein) as to the validity of single-parameter scaling even at $T = 0$ K.

It has subsequently come to the authors attention that Mobius and his co-workers have shown that universal plots of d ln σ/d ln T against ln σ can be made for $Si_{1-x}Cr_x$ (Mobius *et al* 1983, Mobius *et al* 1985) and granular Al (Mobius 1985) on the insulating side of the metal–insulator transition. Single-parameter scaling has also been shown to be consistent with results for ultra-thin $Bi_{14}Te_{11}S_{10}$ crystals (Soonpaa and Schwalm 1982; 1983; 1984), surface conducting layers of cleaved Ge bicrystals (Vul *et al* 1983, Zavaritskaya and Ziyagin 1985) and in p-channel field effect transistors (Dorozlikin *et al* 1987). Most recently Lui *et al* (1990) have shown that single-parameter scaling is also consistent with the results for ultra-thin films of Pd and Bi (evaporated at helium temperatures) in both the weak- and strong-localization regimes.

In this paper, it will be shown that the combined temperature and magnetic field dependent resistivity of disordered bismuth films is also not in conflict with the simple one-parameter AALR scaling. These results are particularly interesting in view of the unconventional behaviour found by McKinnon (1985) for the scaling behaviour of the symplectic models (strong spin–orbit scattering, $B = 0$) and the unitary models ($B \neq 0$), which belong to different universality classes. Moreover, from similar numerical studies Ando and Aoki (1985) claim that single-parameter scaling is invalid for $B \neq 0$, in conflict with former results of Schweitzer *et al* (1984).

2. Theory

The AALR scaling function is defined by

$$\beta = \mathrm{d}\ \ln g/\mathrm{d}\ \ln L \tag{1}$$

where g is the conductance (in units of $e^2/\pi\hbar$) of a subsystem of linear size L in d-dimensional space. White and McLachlan (1986) proposed an experimental 'scaling' function $\beta_{\exp}$, defined by:

$$\beta_{\exp} = \mathrm{d}\ \ln(g/g_0)/\mathrm{d}\ \ln T \tag{2}$$

which is related to the AALR β by

$$\beta_{\exp} = \frac{\mathrm{d}\ \ln(g/g_0)}{\mathrm{d}\ \ln(L/L_e)} \frac{\mathrm{d}\ \ln(L/L_e)}{\mathrm{d}\ \ln T}. \tag{3}$$

The conductance is therefore postulated to be a function of L/L_e, where L_e is the relevant temperature dependent microscopic length of the system (for example, the inelastic diffusion length L_{in} in the weak localization system). g_0 is defined by (White and McLachlan 1986)

$$g_0 = \lim_{\substack{\beta_{\exp}\to 0 \\ T\to 0 \ \text{or} \ T\to\infty}} g(L/L_e). \tag{4}$$

The $T \to 0$ is for a metallic system and the $T \to \infty$ is for a non-metallic (hopping conductivity) system.

Notice that an implicit temperature normalization is made in equation (3). We now also define

$$\beta_N = \mathrm{d}\ \ln g/\mathrm{d}\ \ln(L/L_e) \tag{5}$$

and

$$\beta_{N0} = \lim_{g\to g_0} \beta_N. \tag{5b}$$

With $\Delta\beta_N = \beta_N - \beta_{N0}$, where $\Delta\beta_N$ is now the difference between two points on the 'scaling' curve, using (3) one obtains

$$\Delta\beta_N = \beta_{\exp}[(\mathrm{d}\ \ln L)/(\mathrm{d}\ \ln T) - (\mathrm{d}\ \ln L_e)/(\mathrm{d}\ \ln T)]^{-1}. \tag{6}$$

Now, since L is an arbitrary length imposed upon the system, $\mathrm{d}\ \ln L/\mathrm{d}\ \ln T = 0$. The

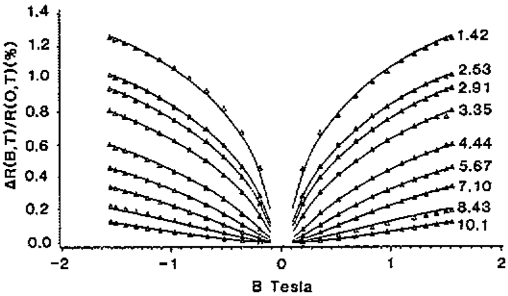

Figure 1. Magnetoresistance $\Delta R(B,T)/R(0,T) = (R(B,T) - R(0,T))/R(0,T)$ versus B for the 20 nm film. The temperature at which each set of measurements are taken is indicated next to the $B = 1.5\ T$ point of each set. The full curves are theoretical data-fits using the theory of Fukuyama and Hoshino (1981), using the parameters given in White and McLachlan (1989). ((L_{in}(1K) = 146 nm, L_0 = 1.5 nm (inelastic scattering length) and L_{SO} = 3 nm (the spin–orbit scattering length).)

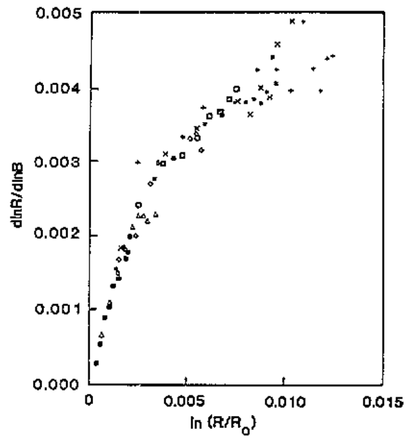

Figure 2. Plot of d ln R/d ln B versus ln R/R_0 for the combined magnetic field and temperature dependence of the 20 nm 0 °C film (see figure 1). Note $R_0 \equiv R(0, T)$. All the data reduces to a single curve (within the error introduced by taking the logarithmic derivatives). The data lies in the temperature range 1.4 K (×) to 7.1 K (⊕), with intermediate temperatures represented by other symbols.

length L_e is defined as a relevant temperature dependent length of the system. In weak (anti)localization, for example, $L_e = L_{in}$, the inelastic scattering length. In the hopping regime $L_e = L_{HOP}$, the mean hopping length.

This approach does not exclude the possibility that universal behaviour, as observed by White and McLachlan (1986), can also be observed in the presence of a magnetic field. The remainder of this paper presents experimental results, which show universal behaviour for a combined temperature and magnetic field dependence.

3. Experimental

White and McLachlan (1986) showed that the TCR data of a wide range of samples could be reduced to a single d ln g/d ln T versus $\ln(g/g_0)$ curve (at $B = 0$). In this paper a similar data reduction is performed for a set of data where there is both a temperature and a magnetic field dependence. The acquisition of this data is described in White and McLachlan (1989) and White (1988). Figure 1 shows the magnetoresistance data for a 20 nm thick polycrystalline bismuth film sputtered onto a glass substrate held at 0 °C. The data in this figure is plotted in the form $\Delta R(B, T)/R(0, T)$ versus B, where $\Delta R(B, T)/R(0, T) = (R(B, T) - R(0, T))/R(0, T)$ and the temperature at which each magnetoresistance curve was measured is indicated on the plot. The full curves are from the theory of Fukuyama and Hoshino (1981) for the magnetoresistance of 2D weak antilocalization. In this regime, the temperature dependence of the inelastic length is given by $L_{in}(T) = L_{in}(1\ \mathrm{K})/T^{(p/2)}$. The values of p, L_{in}(1 K), the spin–orbit scattering

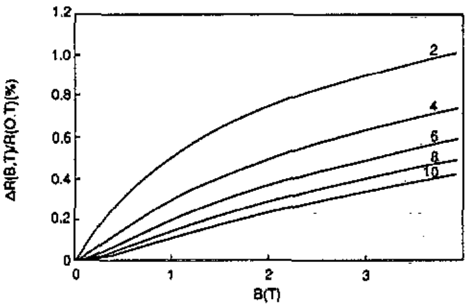

Figure 3. Calculated magnetoresistance curves using the theory of Fukuyama and Hoshino (1981), *with* $L_{in} = (300/T)$ *nm*, $L_{SO} = 3.5$ *nm and* $L_0 = 2.0$ nm, plotted as d ln R/d ln B against ln R/R_0.

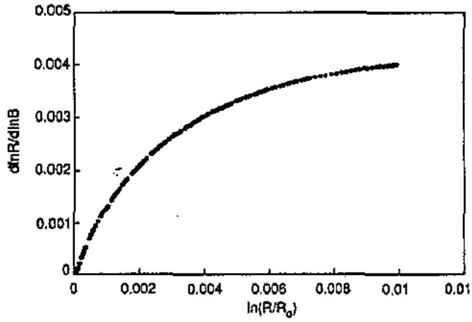

Figure 4. The data in figure 3 plotted in the form d ln R/d ln B against ln R/R_0. The data at the different temperatures now lies on a single curve.

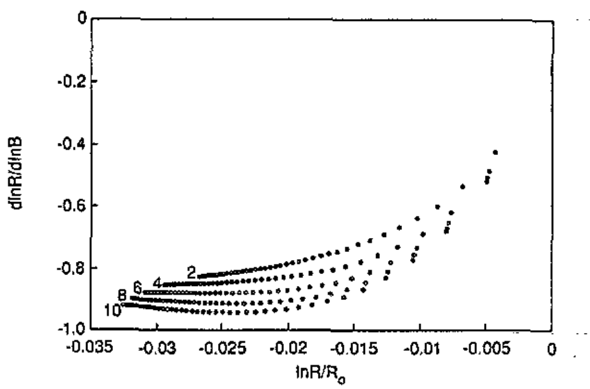

Figure 5. Calculated magnetoresistance curves using the theory of Fukuyama and Hoshino (1981), in the intermediate spin–orbit scattering region, with $L_{in} = (300/T)$ nm, $L_{SO} =$ 350 nm and $L_0 = 2.0$ nm, plotted as d ln R/d ln B against ln R/R_0.

length L_{SO}, the elastic scattering length L_0, and $k_F L_0$ (k_F is the Fermi momentum) are also given in White and McLachlan (1989). k_F was determined from the carrier concentration, which in turn was obtained from the measured Hall coefficient using the semiclassical single-carrier expression relating the carrier concentration to the Hall coefficient.

Figure 2 shows the magnetoresistance data for the same sample plotted in the form d ln $R(B, T)$/d ln B versus ln $R/R_0(R_0 \equiv R(0, T))$. As ρ and R are related by a geometrical factor, a similar curve would be obtained if the data were plotted in the form d ln ρ/d ln B against ln ρ/ρ_0. As can be seen, all the data shown in figure 1 reduced to a single curve when plotted in this way. Similar results were found for a 200 nm polycrystalline bismuth film, which has been shown by White and McLachlan (1988) to be a 3D film.

To see if these results are consistent with the theory of Fukuyama and Hoshino (1981), the theory was used to calculate synthetic magnetoresistance data, at temperatures between 2 and 10 K, which is shown in figure 3. Realistic scattering lengths, given in the caption, which are similar to those found experimentally in thin Bi films were used. When plotted in the form d ln R/d ln B, the synthetic data reduces to a single curve (figure 4). The apparent universality of these results is very tantalizing, especially in view of the remarks at the end of the introduction.

However, it was found when the spin–orbit coupling length was increased from 3.5 to 350 nm that the synthetic data, as shown in figure 5, did not reduce to a single curve. This suggests that in the low spin–orbit scattering or strictly weak-localization case, one-parameter scaling is not consistent with the combined temperature and magnetic field dependence of the conductance. The changes in sign in figure 5 are due to the fact that the synthetic data shown in the figure describes a weak-localization situation and not the weak-antilocalization situation shown in figure 3.

In conclusion, it would appear that single-parameter scaling is consistent with the combined B and T dependence of the conductance in the strong spin–orbit scattering limit, but not in the intermediate spin–orbit scattering regime.

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